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$$\frac{R}{n} + \frac{r}{a/n}, \quad \frac{aR + rn^2}{an}.$$

Placing the first differential coefficient of this expression equal to zero we have

$$\frac{2an^2 r dn - a^2 R dn - an^2 r dn}{a^2 n^2} = 0.$$

From which

$$rn^2 = aR, \quad n^2 = \frac{aR}{r}.$$

Replacing the value of  $a/m$  for one factor in  $n^2$ ,

$$n \frac{a}{m} = \frac{aR}{r}, \quad \frac{n}{m} = \frac{R}{r}, \quad R = \frac{n r}{m}.$$

Or the external resistance equals the total internal resistance. This is seen to be a minimum value for the expression differentiated since the value of the second differential coefficient is greater than zero for the positive value of  $n$ , —the only value it can have.

## THE RADIUS OF THE TERRESTRIAL SPHEROID.

By F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

If there be nothing *new* under the sun, it may not be uninteresting to expand the *old*.

Represent the earth's equatorial radius by  $a$ , the geographical latitude by  $\phi$ , and the geocentric latitude by  $\phi'$ ; then since  $x^2/a^2 + y^2/b^2 = 1$ , we have  $\tan \phi = -dx/dy$ , and  $\tan \phi' = y/x$ . Also, since  $b^2 = a^2(1 - e^2)$ , we have

$$y^2 = a^2(1 - e^2) - (1 - e^2)x^2 \text{ and } y/x = (1 - e^2)\tan \phi.$$

$$\therefore x = \frac{a \cos \phi}{\sqrt{1 - e^2 \sin^2 \phi}} \text{ and } y = \frac{a(1 - e^2) \sin \phi}{\sqrt{1 - e^2 \sin^2 \phi}} \dots (1).$$

Now, the radius of the terrestrial spheroid for any latitude  $\phi$ , is  $\rho = \sqrt{x^2 + y^2}$ .

$$\therefore \rho = a \sqrt{\left(1 - \frac{e^2(1 - e^2) \sin^2 \phi}{1 - e^2 \sin^2 \phi}\right)}, = a \sqrt{[1 - e^2(1 - e^2)(\sin^2 \phi + e^2 \sin^4 \phi)]}.$$

By assuming  $e^2 = 1 - f^2$  and  $\sin^2 \phi = \frac{1}{2}(1 - \cos 2\phi)$ , Encke obtains the series

$$\log \rho = 9.9992747 + 0.0007271 \cos 2\phi - 0.0000018 \cos 4\phi,$$

in which the equatorial radius is unity.

Making  $x = \rho \cos \phi'$  and  $y = \rho \sin \phi'$ , then the former equation of (1) may be written  $e^2 = \frac{\rho^2 \cos^2 \phi' - a^2 \cos^2 \phi}{\rho^2 \sin^2 \phi \cos^2 \phi'}$ ; and by means of this value of  $e^2$ , the elimination of  $e^2$  from the latter equation of (1) may be effected.

$$\therefore \rho^2 \sin^2 \phi' = \frac{(\rho^2 \sin^2 \phi \cos^2 \phi' - \rho^2 \cos^2 \phi' + a^2 \cos^2 \phi)^2}{\rho^2 \sin^2 \phi \cos^2 \phi \cos^2 \phi'}.$$

$$\therefore \rho = a \sqrt{\left( \frac{\cos \phi}{\cos \phi' \cos(\phi - \phi')} \right)} \dots (a).$$

Formula (a) may be deduced in another way; by assuming that

$$e \sin \phi = \sin \psi \dots (2),$$

we obtain

$$\rho \sin \phi' = a(1 - e^2) \sin \phi \sec \psi \dots (\alpha), \text{ and } \rho \cos \phi' = a \cos \phi \sec \psi \dots (\beta).$$

From ( $\alpha$ ) and ( $\beta$ ) by easy deductions,

$$\rho \sin(\phi - \phi') = \frac{1}{2} a e^2 \sin 2\phi \sec \psi \dots (\alpha'), \text{ and } \rho \cos(\phi - \phi') = a \cos \phi \dots (\beta').$$

From (2) and ( $\beta'$ ) we have, respectively,

$$e = \sin \psi / \sin \phi \text{ and } \cos \psi = \rho \cos(\phi - \phi') / a;$$

and after transforming ( $\alpha'$ ), etc., we obtain

$$\rho \sin(\phi - \phi') = a \times \frac{\cos \phi}{\sin \phi} \times \frac{1 - \cos^2 \psi}{\cos \psi} = a \times \frac{\cos \phi}{\sin \phi} \times \frac{a^2 - \rho^2 \cos^2(\phi - \phi')}{a \rho \cos(\phi - \phi')}.$$

$$\therefore \rho^2 = \frac{a^2 \cos \phi}{\sin(\phi - \phi') \cos(\phi - \phi') \sin \phi + \cos^2(\phi - \phi') \cos \phi}.$$

Expanding this denominator, combining terms, etc., we have (a) by a second method of reduction.

In order to obtain formula (a) by a third method, we remember that  $a^2/b^2 = \tan \phi / \tan \phi'$  and that  $x^2 + (a^2/b^2)y^2 = a^2$ ; or after obvious transformations,

$$\rho^2 \cos^2 \phi' + (a^2/b^2) \rho^2 \sin^2 \phi' = a^2, \text{ or } \left[ \cos^2 \phi' + \left( \frac{\sin \phi \times \cos \phi'}{\cos \phi \times \sin \phi'} \right) \sin^2 \phi' \right] \rho^2 = a^2,$$

from which formula (a) is readily deduced.